Voyage: A Journey through our Solar System

Grades 9-12

Lesson 2: The Voyage Scale Model Solar System

On October 17, 2001, a one to ten billion scale model of the Solar System was permanently installed on the National Mall in Washington, DC. The Voyage exhibition stretches nearly half a mile from the National Air and Space Museum to the Smithsonian’s Castle Building. Voyage is a celebration of what we know of Earth’s place in space and our ability to explore beyond the confines of this tiny world. It is a celebration worthy of the National Mall. Take the Voyage at www.voyageonline.org, and consider a Voyage exhibition for permanent installation in your own community.

This lesson is one of many grade K-12 lessons developed to bring the Voyage experience to classrooms across the nation through the Journey through the Universe program. Journey through the Universe takes entire communities to the space frontier.

Voyage and Journey through the Universe are programs of the National Center for Earth and Space Science Education, Universities Space Research Association (www.usra.edu). The Voyage Exhibition on the National Mall was developed by Challenger Center for Space Science Education, the Smithsonian Institution, and NASA.

Copyright June 2006
Lesson 2: The Voyage Scale Model Solar System

Lesson at a Glance

Lesson Overview
It is challenging to design a scale model of the Solar System where the same scale is used to portray not only the physical sizes of the Sun and planets, but also the distances between them. Planets are tiny worlds in a vast space. In October 2001, the Voyage Scale Model Solar System opened in Washington, DC, displaying a one to ten billion scale of the sizes of the Sun and planets, and the distances between them. In this lesson, students will replicate the Voyage model to experience the size of the Solar System.

Lesson Duration
Two 45-minute class periods

Core Education Standards

National Science Education Standards
Standard E1: Abilities of technological design
- Propose designs and choose between alternative solutions: Students should demonstrate thoughtful planning for a piece of technology or technique. Students should be introduced to the roles of models and simulations in these processes.

AAAS Benchmarks for Science Literacy
Benchmark 12B1:
- Use ratios and proportions, including constant rates, in appropriate problems.
Related Education Standards

AAAS Benchmarks for Science Literacy

Benchmark 9A1:
- Comparison of numbers of very different size can be made approximately by expressing them as nearest powers of 10.

Benchmark 11B1:
- The basic idea of mathematical modeling is to find a mathematical relationship that behaves in the same ways as the objects or processes under investigation. A mathematical model may give insight about how something really works or may fit observations very well without any intuitive meaning.

Essential Question

- How is a scale model useful in demonstrating the sizes and distances of objects in the Solar System?

Concepts

Students will learn the following concepts:
- Models are powerful tools of exploration.
- The sizes of the planets and distances between the planets in the Solar System span a large range.
- A good scale model fits within humanly manageable sizes.

Objectives

Students will be able to do the following:
- Design a scale model Solar System for public display.
- Describe the characteristics of the Solar System that make construction of a good scale model so difficult.
Science Overview

Voyage is a 1 to 10-billion scale model of the Solar System that was permanently installed in Washington, D.C., in October 2001. The real Solar System is exactly 10 billion times larger than the Voyage model. On this scale the Sun is about the size of a large grapefruit. The Earth is 15 meters (50 feet) away and smaller than the head of a pin. The entire orbit of the Moon fits comfortably in the palm of a child’s hand. Pluto, the farthest planet, is 600 meters (2,000 feet or 6.7 football fields) away from the Sun. The nearest star to the Sun would be the size of a cherry in California. In this lesson, students will design a scale model Solar System that will turn out to have a scale very close to that of Voyage.

Review of Scale Models

In a scale model, all linear sizes and distances are reduced or enlarged by the same factor, so as to be in the same proportion to the original. For example, in a 1:20 scale model car, the length of the car, the width of the car, the height of the car, the diameter of the steering wheel, and the thickness of the head rests are all 1/20 of the corresponding sizes in the real car. (Note that areas and volumes will not be scaled by a factor of 1/20.)

In a scale model, the ratio of a model size to the original size is the same for all parts of the model. For example, if you have a scale model airplane, the ratio of the length of the real airplane to the length of the model is the same as the ratio of the length of the real airplane’s wings to the length of the model plane’s wings. The ratios can be written as

\[
\frac{\text{model airplane length}}{\text{real airplane length}} = \frac{\text{model wing length}}{\text{real wing length}} = \frac{\text{model tire diameter}}{\text{real tire diameter}} = \frac{\text{model window height}}{\text{real window height}} = \ldots
\]

Any one of those ratios tells the scale of the model.

In a scale model, the quantities that are to scale include distances and sizes, but not areas and volumes. Area will vary as the square of the linear dimension, and the volume will vary as the cube of the linear dimension. For example, if you have a model airplane that is a 1:100 scale model of the real thing, its length is 1/100 the size of the original, its surface area is 1/10,000 that of the original, and its volume is 1/1,000,000 that of the original.
Solar System Sizes and Distances

Sizes and distances of Solar System objects are shown in the Student Worksheet. Note the wide variation. Pluto is more than 100 times farther from the Sun than is Mercury. Pluto’s distance from the Sun is more than 15,000 times the Earth-Moon distance. Therefore, it is not easy to make a small scale-model Solar System that shows the locations of the Sun and Pluto, as well as the Earth and Moon. Portraying the sizes of the planets on the same scale as the distances to the Sun is even more problematic. For example, the Sun-Pluto distance is 2.5 million times the diameter of Pluto. It is, therefore, clear that it is not possible to show the sizes of the planets as well as their distances from the Sun on a small model.
Conducting the Lesson

Warm-Up & Pre-Assessment

Teacher Materials

- Various models (e.g., model airplanes, cars, trains, molecules, maps, globe)
- Solar System posters
- (Optional) Simple geometric shapes (sphere, cube, etc.)

Preparation & Procedures

1. Have some scale models (e.g., model airplane, model car, map, globe) on hand and, if available, some simple geometric shapes (sphere, cube, etc.) to practice calculating ratios in scale models.

2. Review characteristics of scale models with students. All linear dimensions (length, width, height, diameter, etc.) are in the same proportion to the original.

3. Ask for examples. One example might be an architect’s scale model of a sports arena. If it is a 1:50 scale model, then the model’s height, length, and width are all 1/50 of the size of the real arena.

4. Referring to examples from Step 3, ask what quantities are to scale. Distance? Size? Area? Volume? (Desired answer: only distance and linear dimensions (e.g., width), but not areas or volumes.)

Teaching Tip

To help students distinguish between non-scale models and scale models, show examples. A model car might be to scale, but a relief globe (with bumps in the mountainous regions) is not to scale; the mountains would not be that high if it were to scale. In fact, you wouldn’t be able to feel them at all, because the Earth is so big compared to the size of the mountains.
5. Ask students the following question: In the movie “The Amazing Colossal Man,” a man grows to 10 times the size of a normal person. Why would such a giant not be able to stand up? (Desired answer: assuming everything is in proportion and the man’s mass increases during the process so that his density remains the same, if the man’s height is 10 times normal, then his volume (and, therefore, weight) will be $10^3$—or 1,000—times normal. He wouldn’t be strong enough to stand up. Additional (optional) information: one might estimate that the strength of bones and muscles is proportional to their cross-sectional area. Thus, the amazing colossal man’s bones and muscles are $10^2$ (or 100) times as strong as a normal man’s, but that won’t be enough to deal with his 1,000-times-normal weight.)
Activity: An Outdoor Scale Model Solar System

In this activity, students calculate an appropriate scale from which to create a scale model Solar System. They decide the sizes and distances of all of the planets in order to create their model.

Student Materials (per pair of students)

- Calculator
- Pencil
- Paper
- Metric ruler with mm divisions

Student Materials (per student)

- Student Worksheet

Preparation & Procedures

1. Have students work in pairs or small groups.

Part A: Create the Model

2. Explain that a model is most useful if a person can hold and manipulate it (e.g., about 30 cm). That is why a typical globe, found in many classrooms, is about 30 cm (one foot) in diameter. But what if a model has many parts of all different sizes? They cannot all be about 30 cm in size. So, a good compromise is to select a scale where the largest part of the model is $x$ times larger than 30 cm, while the smallest part is that same number $x$ times smaller than 30 cm. For example, if the largest part of the model is 25 times the size of the smallest part, it would be good to scale the model so that the largest part has a size of about 1.5 m (5 ft), while the smallest part is about 6 cm (1/5 ft).

3. Discuss ratios using the information in Box 1.

4. Distribute Student Worksheet.

5. Tell the students that they are in charge of designing a Solar System model to be displayed in their state’s capital. They should design the model so that the largest size or distance (in this case, the distance from the Sun to Pluto) is about as many times larger than 30 cm (one foot) as the smallest size (in this case, the diameter of Pluto) is smaller than 30 cm (one foot); see Box 1.
A Convenient Size

A small globe might have a diameter of 30 cm. While that is a convenient size for a model, a scale model of the Solar System could not have everything that size. So, what scale should be used? Pick the scale so that the ratio of the largest size ($LS$) to 30 cm is the same as the ratio of 30 cm to the smallest size ($SS$):

$$\frac{LS}{30 \text{ cm}} = \frac{30 \text{ cm}}{SS} \quad (1)$$

What Scale?

If you know how big you want the largest size ($LS$) in the model to be, then the scale is the ratio of the true largest size ($TLS$, e.g., the distance from the Sun to Pluto) to the size in the model ($LS$):

$$scale = \frac{TLS}{LS} \quad (2)$$

The scale is also the ratio of the true smallest size ($TSS$, e.g., Pluto’s diameter) to the model’s smallest size ($SS$):

$$scale = \frac{TSS}{SS} \quad (3)$$

You can solve Eq. 2 for $LS$, and you can solve Eq. 3 for $SS$. Then, plug those formulae for $LS$ and $SS$ into Eq. 1, and solve for the scale:

$$scale = \frac{\sqrt{(TLS)(TSS)}}{30 \text{ cm}} \quad (4)$$

Caution: When calculating ratios, make sure you use the same units for both the numerator and denominator.
6. Have the students complete steps 1-2 in the Student Worksheet.

7. Verify that their scale is near 10 billion (the scale for the *Voyage* model). They may get closer to 8 billion; that would be all right.

**Part B: Use the Model**

8. Have students answer questions 3-5 in the Student Worksheet using their scale model, when necessary.

**Teaching Tip**

- Dividing the real size by the scale gives the model size. Or, if you know the model size, multiplying it by the scale gives the real size.
- Make sure that the students convert units correctly. The data table provides real sizes and distances in km. Applying the scale will provide model distances in km. The Student Worksheet asks for model sizes in mm and distances in meters.

**Reflection & Discussion**

Discuss the following questions with students:

- Why is a model necessary to understand the relative positions of objects in the Solar System?
- What else can be learned about the Solar System from a scale model?
- What cannot be learned about the Solar System using a scale model?
- To what extent do the illustrations of the planets (on the Student Worksheet) depict their relative sizes and distances accurately?
- Why was the scale chosen so that the largest size is a certain number \( x \) times larger than 30 cm (1 ft) and the smallest size is the same number \( x \) times smaller than 30 cm (1 ft)?

**Transfer of Knowledge**

- The Milky Way Galaxy has a diameter of 120,000 light-years. (One light-year = \( 9.5 \times 10^{12} \) km) Would it be difficult to make a scale model of the Milky Way Galaxy showing both the size of the Sun (diameter = \( 1.4 \times 10^8 \) km), as well as its distance from the center of the galaxy (30,000 light-years)?
- How crowded is space? Are astronomical objects fairly close together or very spread out? Are planets or asteroids likely to collide frequently? Why or why not?

When using Equation 4 in Box 1, express the true largest size (TLS) and true smallest size (TSS) in cm. Then the units will cancel out. The scale is dimensionless (i.e., has no units).
Placing the Activity Within the Lesson
It is not easy to make a scale model Solar System because of the enormous range of sizes and distances. In this activity, students find a reasonable scale so that the small sizes of planets, as well as the large distances, are visible. They learn about ratios and scale models, and then they use their model to learn about the Solar System. Possible questions to ask the students include:

- Why is it difficult to make a scale model Solar System? (Desired answer: because of the enormous range of sizes and distances)
- Once we have a scale model Solar System, are all the sizes and distances easy to see? (Desired answer: no, but this scale is the most manageable scale given the limitations of what we can easily see)
- Suppose Pluto was a convenient size of 30 cm (1 foot) in diameter. How far would Pluto be from the Sun in that model? Would that be a good distance for a scale model Solar System? (Desired answer: if the model Pluto were 30 cm in diameter, then the model distance from the Sun to Pluto would be 775 km—about 450 miles! Obviously, that would not be a good size for a model, because we wouldn’t be able to see the whole model at once.)

Lesson Adaptations
Talented and Gifted: Ask the students the following questions-
- Suppose a spacecraft actually travels through the Solar System at a speed of 48,000 km/hr (30,000 miles/hour). How fast would it travel in your scale model? (Answer: 3.8 mm/hr)
- The Earth moves in its orbit around the Sun at a speed of 67,000 miles/hour (110,000 km/hr). How fast does the Earth move in your scale model? (Answer: 8.8 mm/hr)
- The speed of light is \(3 \times 10^8\) meters/second. How fast would light move in your model? (Answer: 2.4 cm/s)

Special Education: Ask the students the following questions-
- If you have a 1:50 scale model of a car, how many model cars would you have to put end-to-end to be as long as the real car? (Answer: 50)
- If you have a 1:10,000,000,000 scale model of the Solar System, how many model Earths would you have to put side-by-side to be as big as the real Earth’s diameter? (Answer: 10,000,000,000)
ASSessment CriTeria for ActiVity

Students’ progress in this activity may be assessed during the Reflection & Discussion and/or Transfer of Knowledge stages, by observing if they can

- Determine scales and use them to calculate distances in a scale model.
- Get a sense for how spread-out things are in the Solar System.
- Explain the uses and limitations of scale models.
Lesson Wrap-Up

Lesson Closure
A nice follow-up activity after designing the scale model Solar System is to go outside and actually lay it out. Students can make a sign for each planet showing the name of the planet in large letters (visible from other distant planets) as well as the size of the planet drawn to scale (which, for small planets, may be just a dot). Students can then see, firsthand, just how large the distances in the Solar System are compared to the size of the planets. Discuss what they can find out about the Solar System by creating the model. For example, how big does the Sun appear from the various planets? How easy or hard is it to see one planet while standing on another planet? Have students compare their model with the Voyage scale model Solar System on the National Mall in Washington, DC. (They can visit the Voyage web site: http://www.voyageonline.org/) Are the scales similar? Will any students be able to visit the Voyage model any time soon?

Notes:
Resources

Internet Resources & References
Student-Friendly Web Sites:
Astronomical Picture of the Day
antwrp.gsfc.nasa.gov/apod/
NASA: Planetary Photojournal
photojournal.jpl.nasa.gov/
The Nine Planets Website
www.nineplanets.org/
Star Date
stardate.org/resources/ssguide/
Voyage Online
www.voyageonline.org/

Teacher-Oriented Web Sites:
AAAS Benchmarks for Science Literacy
www.project2061.org/tools/benchol/bolintro.htm
The Busy Teacher’s Web Site
www.ceismc.gatech.edu/busyt/astro.html
Exploring Planets in the Classroom
www.spacegrant.hawaii.edu/class acts/
NASA: Planetary Photojournal
photojournal.jpl.nasa.gov/
NASA: Planetary Sciences at the National Space Science Data Center
nssdc.gsfc.nasa.gov/planetary/
NASA Quest
quest.arc.nasa.gov/sso/teachers/
National Science Education Standards
www.nap.edu/html/nses/
The Nine Planets
www.nineplanets.org/
Star Date
stardate.org/resources/ssguide/
Voyage Online
www.voyageonline.org/
Teacher Answer Key

1. \[ \text{scale} = \frac{\sqrt{(5.87 \times 10^{14} \text{ cm}) \times (2.40 \times 10^8 \text{ cm})}}{30 \text{ cm}} = 1.25 \times 10^{10} \]

2. Model Sun diameter:
   \[ \text{model size} = \frac{\text{real size}}{\text{scale}} = \frac{1.4 \times 10^{11} \text{ cm}}{1.25 \times 10^{10}} = 11.2 \text{ cm} \]

<table>
<thead>
<tr>
<th>PLANET</th>
<th>Model Size (Diameter in MM)</th>
<th>Model Distance to Sun (In M)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mercury</td>
<td>0.39 mm</td>
<td>4.63 m</td>
</tr>
<tr>
<td>Venus</td>
<td>0.97 mm</td>
<td>8.66 m</td>
</tr>
<tr>
<td>Earth</td>
<td>1.0 mm</td>
<td>12.0 m</td>
</tr>
<tr>
<td>Mars</td>
<td>0.54 mm</td>
<td>18.2 m</td>
</tr>
<tr>
<td>Jupiter</td>
<td>11 mm</td>
<td>62.3 m</td>
</tr>
<tr>
<td>Saturn</td>
<td>9.7 mm</td>
<td>115 m</td>
</tr>
<tr>
<td>Uranus</td>
<td>4.1 mm</td>
<td>230 m</td>
</tr>
<tr>
<td>Neptune</td>
<td>4.0 mm</td>
<td>360 m</td>
</tr>
<tr>
<td>Pluto</td>
<td>0.19 mm</td>
<td>470 m</td>
</tr>
</tbody>
</table>

3. 

<table>
<thead>
<tr>
<th>OBJECT</th>
<th>Model Size (In MM)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Earth’s moon</td>
<td>0.28</td>
</tr>
<tr>
<td>Ganymede</td>
<td>0.42</td>
</tr>
<tr>
<td>Ceres</td>
<td>0.077</td>
</tr>
</tbody>
</table>

4. 8.0 cm

5. 3,254 km (about half the real Earth’s radius)

Teaching Tip

- Notice that the units (cm) cancel out. That’s because this scale is really a ratio of two distances (or sizes) with the same units.
- Dividing the real size by the scale gives the model size. Or, if you know the model size, multiplying it by the scale gives the real size.
You are in charge of designing a Solar System model to be displayed in your state’s capital. You should design the model so that the ratio of the largest size (the distance from the Sun to Pluto) to 30 cm (1 ft) is equal to the ratio of 30 cm to the smallest size (the diameter of Pluto).

A Convenient Size

A small globe might have a diameter of 30 cm. While that is a convenient size for a model, a scale model of the Solar System could not have everything that size. So, what scale should be used? Pick the scale so that the ratio of the largest size ($LS$) to 30 cm is the same as the ratio of 30 cm to the smallest size ($SS$):

$$\frac{LS}{30 \text{ cm}} = \frac{30 \text{ cm}}{SS} \quad (1)$$

What Scale?

If you know how big you want the largest size ($LS$) in the model to be, then the scale is the ratio of the true largest size ($TLS$, e.g., the distance from the Sun to Pluto) to the size in the model ($LS$):

$$scale = \frac{TLS}{LS} \quad (2)$$

The scale is also the ratio of the true smallest size ($TSS$, e.g., Pluto’s diameter) to the model’s smallest size ($SS$):

$$scale = \frac{TSS}{SS} \quad (3)$$

You can solve Eq. 2 for $LS$, and you can solve Eq. 3 for $SS$. Then, plug those formulae for $LS$ and $SS$ into Eq. 1, and solve for the scale:

$$scale = \frac{\sqrt{(TLS)(TSS)}}{30 \text{ cm}} \quad (4)$$

Caution: When calculating ratios, make sure you use the same units for both the numerator and denominator.
### Sizes and Distances in the Solar System

<table>
<thead>
<tr>
<th></th>
<th>Distance from Sun (AU)</th>
<th>Distance from Sun (10^6 km)</th>
<th>Diameter (km)</th>
<th>Diameter (Earth diameters)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sun</td>
<td>0.0</td>
<td>0.0</td>
<td>1,400,000</td>
<td>109</td>
</tr>
<tr>
<td>Mercury</td>
<td>0.39</td>
<td>57.9</td>
<td>4,900</td>
<td>0.38</td>
</tr>
<tr>
<td>Venus</td>
<td>0.72</td>
<td>108.2</td>
<td>12,100</td>
<td>0.95</td>
</tr>
<tr>
<td>Earth</td>
<td>1.00</td>
<td>149.6</td>
<td>12,800</td>
<td>1.00</td>
</tr>
<tr>
<td>Mars</td>
<td>1.52</td>
<td>227.9</td>
<td>6,800</td>
<td>0.53</td>
</tr>
<tr>
<td>Jupiter</td>
<td>5.20</td>
<td>778.6</td>
<td>143,000</td>
<td>11.21</td>
</tr>
<tr>
<td>Saturn</td>
<td>9.58</td>
<td>1434</td>
<td>121,000</td>
<td>9.45</td>
</tr>
<tr>
<td>Uranus</td>
<td>19.20</td>
<td>2872</td>
<td>51,000</td>
<td>4.01</td>
</tr>
<tr>
<td>Neptune</td>
<td>30.05</td>
<td>4495</td>
<td>50,000</td>
<td>3.88</td>
</tr>
<tr>
<td>Pluto</td>
<td>39.24</td>
<td>5870</td>
<td>2,400</td>
<td>0.19</td>
</tr>
</tbody>
</table>

### Moon (Earth’s moon)

<table>
<thead>
<tr>
<th>Distance from Earth</th>
<th>Diameter</th>
</tr>
</thead>
<tbody>
<tr>
<td>384,000 km</td>
<td>3,476 km = 0.27 Earth diameters</td>
</tr>
</tbody>
</table>

### Ceres (an asteroid)

<table>
<thead>
<tr>
<th>Average distance from Sun</th>
<th>Diameter</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.8 AU</td>
<td>960 km = 0.075 Earth diameters</td>
</tr>
</tbody>
</table>

### Ganymede (one of Jupiter’s moons)

<table>
<thead>
<tr>
<th>Distance from Jupiter</th>
<th>Diameter</th>
</tr>
</thead>
<tbody>
<tr>
<td>1,070,000 km</td>
<td>5,262 km = 0.41 Earth diameters</td>
</tr>
</tbody>
</table>

1. Calculate the scale for your model:
2. Using this scale, and the data in the Solar System Data table, determine the sizes and distances of the Sun and planets in the model. Be careful to handle unit conversions correctly.

Model Sun diameter (in cm): _____________________

<table>
<thead>
<tr>
<th>PLANET</th>
<th>MODEL SIZE (DIAMETER IN MM)</th>
<th>MODEL DISTANCE TO SUN (IN M)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mercury</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Venus</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Earth</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mars</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Jupiter</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Saturn</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Uranus</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Neptune</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Pluto</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

3. Calculate the size in your model of Earth’s moon, Jupiter’s moon Ganymede, and the asteroid Ceres.

<table>
<thead>
<tr>
<th>OBJECT</th>
<th>MODEL SIZE (IN MM)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Earth’s moon</td>
<td></td>
</tr>
<tr>
<td>Ganymede</td>
<td></td>
</tr>
<tr>
<td>Ceres</td>
<td></td>
</tr>
</tbody>
</table>

4. In the asteroid belt (between the orbits of Mars and Jupiter), there are many asteroids (big chunks of rock) of many different sizes. Some of them are about 1 km in diameter. The distance between one of these asteroids and its nearest 1-km-diameter neighbor is typically about 10⁶ km. How far away is that in your model?

Model distance between two asteroids (in cm): _____________________

5. The nearest star to the Sun is Proxima Centauri, located 4.3 light-years from the Sun. (One light year = 5.88 trillion miles = 9.46 x 10¹² km) How far away would Proxima Centauri be in your model?

Model distance to Proxima Centauri (in km): _____________________